A Place-Based Approach to the Praxis of Ethnomathematics Education: Investigating Woven Bamboo *Doko* of Southern Bhutan

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Abstract

This study investigated the ethnomathematical educational potential in the weaving and use of traditional bamboo baskets, locally known as *Dok*o, that are commonly found in southern Bhutan. Data collection was completed in two rural village communities where these baskets are locally produced and used. Village elders who are expert *Doko* weavers and community members who commonly use *Doko* were interviewed following a semi-structured approach. Ethnographic observations and visual content analysis were used to investigate the anatomy of the baskets. Thematic analysis was used to analyse interview data. Key findings of the study reveal the presence of ethnomathematical concepts including number patterns, quadrilaterals, angular developers, truncated pyramidal morphologies, and tessellations woven into the structure of the *Doko*. Additional thematic analysis demonstrates how the ecological, cultural, and economic significance of *Doko* link the practice of weaving and using these baskets to the daily life of village communities. Reviewed holistically, we share how *Doko* could serve as an ethnomathematical place-based asset to help support Bhutan's unique goals of cultural promotion and preservation through Educating for Gross National Happiness.

Key words: *Doko*, community, ethnomathematics, patterns, place-based-education.

Introduction

"A man is born in a bamboo cradle and goes away in a bamboo coffin"
(Nirala, et al, 2015).

Spanning birth and death, bamboo can hold ecological, cultural, and economic significance for many cultures across the globe. Recently, traditional products woven from bamboo have been increasingly recognized for their ethnomathematical significance (Maryati & Prahmana, 2019; Muhtadi et al., 2017; Yudianto et al., 2020). Building on this trend, this study examines the ethnomathematical educational potential of woven bamboo items known as *Doko* (Figure 1). These baskets, which have yet to receive much documentation, are traditionally crafted and used in the southern Himalayan foothills of Bhutan.

Figure 1: An elder basket weaver from the village in Ahaley, Bhutan. A traditional bamboo basket known locally as a *Doko(*डोको) can be observed hanging from his right shoulder.

The stories of bamboo baskets are inherently grounded in stories of weaving. Through this work, we attempt to weave multiple descriptive threads including the challenges faced by modern mathematics education, Bhutan's legacy of ethnic diversity, and the practice of place-based approach to learning for the preservation of culture. What emerges is a narrative where ethnomathematics builds a story of learning and connection. We invite readers to consider how *Doko* can share these stories. To do so, requires an integrated reflection on these interrelated concepts.

Ethnomathematics in Bhutan – Opportunities and Challenges

The field of ethnomathematics rests at the intersection between culture and mathematics. According to D`Ambrosio (2001) the term "ethno" encompasses the multitude of practices that make up the cultural identity of a group. These elements can include language, codes, values, beliefs, food and dress, habits, and physical traits. Ethnomathematics seeks to explore the relationship that cultural elements share with broad mathematical concepts (i.e arithmetic, classifying, ordering, inferring, and modelling). In other words, ethnomathematics examines how different cultural groups use mathematics. Accordingly, the study of ethnomathematics provides a direct pathway for investigating cognitive patterns and traditional customs.

Far-reaching in its potential, two identified strengths of ethnomathematics reside in its capacity for educational improvement and cultural preservation (Rowlands & Carson, 2002). Across various cultural contexts, ethnomathematics been shown to increase learner engagement and metacognitive mathematical performance (Herawaty et al., 2018; Imswatama & Lukman, 2018). Recent work has additionally shown how these outcomes may even be enhanced when practiced in an outdoor learning environment (Widada et al., 2019). Concurrently, as students and educators engage with learning via ethnomathematics, they are actively working to build understanding of traditional and living cultures often endemic to their place (Abdullah, 2017; D'Ambrosio, 2007).

Such scholastic and cultural outcomes are identified as increasingly desirable in the evolving standard of 21st century education (Gravemeijer et al., 2017). However, challenges stemming from the ever-increasing pressures of globalization and emphasis on integrated STEM learning continue to act as educational barriers (English, 2015). The perpetuation of traditional and Eurocentric approaches to mathematics education can further exacerbate these challenges, particularly for developing nations (Brandt & Chernoff, 2015). Though ethnomathematics offers one promising approach to address these issues, its practice remains limited in many contexts.

The lack of prevalent ethnomathematics practice is likewise the case in the nation of Bhutan. This limited implementation of ethnomathematics can likely be linked to the developmental trajectory of the nation's educational system. A complex suite of economic, ecological, and cultural factors has shaped the nation's pathway towards developing a modern educational system. We outline this system in the following paragraphs.

Traditionally, Bhutan's educational systems was grounded within the country's monastic centers. Until the 1950's, this monastic education had been responsible for the formal culture and spiritualization of all Bhutan's youth. With recognition of the emerging movement towards globalization, the Third Druk Gyalpo (King) Jigme Dorji Wangchuck ushered the nation towards the development of a of Western-styled 'modern' educational system. This transition centered around the adoption and importation of customs, texts, and human resources from the neighboring nation of India. In fact, it has only been in the last 20 years that the Bhutanese education system has become largely independent from India. (Namgyel & Rinchhen, 2016).

As a geographically and (until the 1960's) culturally isolated Himalayan Kingdom, Bhutan is perhaps most well-known for its developmental vision of Gross National Happiness (GNH). Articulated first in the 1970's by the Fourth King Jigme Singye Wangchuk (Planning Commission, 1999), this developmental trajectory has sought to balance the nation's economic growth alongside environmental and cultural preservation through the practice of good governance (Wangmo & Valk, 2012).

Working to break from early reliance on Indian-based curricula and teaching staff (Gyamtso & Maxwell, 2012), Bhutan began a nation-wide educational reform effort in 2008 called Educating for Gross National Happiness (Hayward & Colman, 2010). This Educating for GNH initiative attempts to provide Bhutanese educators with the ways and means to integrate GNH values into their teaching. These values, with roots stemming from teachings central to Buddhist philosophy, include overarching concepts such as mindfulness, empathy, non-alienation, and responsible social leadership. Importantly under Educating for GNH, these values are seen as essential educational outcomes alongside traditional knowledge and content.

A set of philosophical pillars, or developmental objectives, are articulated to structure the implementation of GNH (Sherab, 2013). The preservation and promotion of culture serves as one of these foundational pillars. Accordingly, there has been an increased emphasis on establishing culturally relevant teaching practices within Bhutanese schools. Primed with a revitalized focus on cultural promotion and preservation, the current educational climate in Bhutan presents a distinct scenario that could benefit from the integration of ethnomathematics in everyday teaching and learning.

Despite an educational focus on cultural promotion of preservation, limitations in resources, support, and training, have presented barriers for many Bhutanese educators to actualize this vision (Powdyel, 2014). These barriers persist especially within the discipline of mathematics. Dolma (2017, p. 23.) shares how Bhutanese teachers perceive that they lack modern approaches for teaching mathematics in alignment with the contemporary reformed educational vision.

Interestingly, it appears that ethnomathematics is a practice strongly aligned with the Bhutanese GNH vision of education. However, despite this alignment, implementation of formal ethnomathematics remains limited. The social and cultural inertia from an early importation of a foreign educational system is likely factor for this trend. Yet there is increasing interest to incorporate locally sourced cultural systems into mathematics teaching practice. Therefore, identifying pedogeological approaches that can bridge this gap has emerged as an important mission within the education sector.

Transitioning Towards Place-Based Education

Increasingly, many educators in Bhutan are turning towards the concept of place-based education (PBE) as one method to address these challenges (Dorji et al., 2021). While multiple definitions of PBE have recently emerged, in its most basic sense PBE is an approach that connects learning to communities to increase student and teacher engagement, educational outcomes, and community impact (Smith, 2017; Thapa et al., 2013). In the long-term, place-based approaches can lead to an increase in student agency, community connections and equity – in that *all* students see themselves as possessing the necessary skills and resources to be valuable members of a thriving community (Vander Ark et al., 2020).

Central to effective implementation of PBE is a critical consideration of place as a multidimensional human construction (Ardoin, 2006). In particular, place-based practitioners have worked to situate the concept of place within the traditional dimensions of sustainability: local ecology, culture, and economy (Vander Ark et al., 2020). Meaningful and critical inquiry into these elements of place is argued to rest at the foundation of successful PBE (Gruenewald, 2003).

The identification and use of community assets to structure place-based learning is one common approach to implement PBE (Vander Ark et al., 2020). Engaging with these assets, which can include local experts, objects, and experiences, is often viewed as a necessary for educators to put place-based teaching into practice. Currently, Bhutanese educators are struggling at this particular juncture of identifying local community assets to situate place-based learning. It is at this point where an ethnomathematics approach may hold a well oriented potential for the Bhutanese educational reform.

As a landlocked country located on the Eastern edge of the Himalayan range, the nation of Bhutan is relatively small in geographic extent. Despite its relatively small size, however, Bhutan is a country of immense cultural diversity. More than twenty-eight languages are actively spoken within the country. Each language carries its own traditional mathematics as manifested in music, visual arts and counting systems. A significant number of local dialects give rise to local variability in the language use. These various languages are spoken amongst six primary ethnic groups. While modern socio-political reorganization has begun to demographically homogenize many parts of the country, each ethnic group traditionally hails from a particular geographic region within Bhutan. The six primary ethnic groups are as follows: Ngalop (western and northern Bhutan); Sharchop (eastern Bhutan); Lhotshampa (southern Bhutan); Tibetan (western and northern Bhutan); Indigenous Ethnic Groups (Lepcha, Brokpa, Doya and Oraon tribes -spread throughout Bhutan) and Ancient Tribal Populations (Lhokpu, Mönpa, and Gongduk – spread in southwestern, east-central Bhutan).

With schools and mathematics educators now located throughout each of these traditional cultural regions in Bhutan, opportunities for ethnomathematics education are likely plentiful. However, to date, there has been limited emphasis on building ethnomathematical capacity for these educators. This paper seeks to provide a model of ethnomathematics investigation into community cultural assets for their educational potential. Through examination and discussion, we further examine how a place-based approach to ethnomathematics offers a potential framework to address Bhutan's educational challenge. To situate this investigation, our team choose to focus the ethnomathematics investigation on the woven bamboo baskets of southern Bhutan, traditionally known as *Doko*.

Study Area and an Overview of the Doko (Traditional Lohtsampa Baskets)

This study focused its investigation on the cultural traditions of communities situated within the Samtse Dzongkhag, the southwestern most district of Bhutan. Located within the Himalayan foothills are many villages with predominant Lhostampa populations. These communities are typically small and rural. Traditional crafts and extractive industry (forestry/mining) provide some income for these communities, however local economic conditions are dominantly based in subsistence or small-farm agricultural production. While mechanization and modernization of agricultural techniques has become increasingly widespread in Bhutan, the majority of agricultural practices are still done through traditional methods like hand planting and harvesting. Accordingly, the use of traditional farming and household objects, like bamboo *Doko (*डोको in Nepali), is common.

Two small villages of were selected for detailed investigation in this study. The communities at Ahaley (locally known as *Damshangma*) and Tamang Dara (locally known as *Tsherina*) are representative of the typical farming communities described below (Figure 2). Selection of these villages was dictated by their proximity to the district's capital, in addition to the identified community members with traditional knowledge of the *Doko* products (see methods for more details).

Figure 2: A view to the west overlooking the village of Tamangdara (locally known as *Tsherina*). Note the agricultural production in the image foreground that is typical of communities in the southern Bhutanese foothills.

Doko (डोको in Nepali) is a typical bamboo basket, normally in a truncated pyramidal shape (Figure 3). *Doko* is traditionally derived from Nepali culture, but this culture is prevalent in southern Bhutan and the Smatse area. The baskets are commonly used for household and agricultural purposes (Gurung, 2020). However, they are also used extensively in family rituals and cultural celebrations, building a sense of multidimensional importance.

Till date, little has been carried out in terms of documentation, research and teaching in these areas.

(a) (b)

Figure 3: Examples of woven bamboo *Doko*: (**a**) typical styled *Doko* with view of the upturned basket base, a metric measuring tape is visible the left side of the basket for scale; and (**b**). village farmer using a traditional head strap to carry a loaded *Doko.*

Our study sought to investigate the ethnomathematical educational potential of these woven bamboo *Doko.* The following broad research questions were designed to direct this study:

- What mathematical concepts are embedded in the process and structure of a woven *Doko*?
- What ecological, cultural, and economic relationships are linked to the practice of weaving and using *Doko* in the local Samtse communities?
- How might the practice and presence of a woven *Doko* support teaching and learning mathematics in a Bhutanese school?

Methods

This study adopted a qualitative-descriptive approach informed by a constructivist worldview (Creswell, 2013). Our process sought to situate data collection within our research participants' natural setting. Likewise, we framed our data analysis through a combination of inductive coding and a-priori thematic interpretations. Concurrently, these interpretations were constructed with an awareness that we as the researchers were attempting to make sense of our own contextual and educational experiences (Cresswell, 2009; Creswell, 2014).

There is a marked scarcity of prior research on the weaving and uses of *Doko* in a Bhutanese context. Accordingly, we recognized the need for recording authentic perspectives from individuals actively involved with the production and use of these *Doko*. Examining both the symbolic significance and cultural interpretations of the *Doko* required an approach where we could learn about, record and ultimately portray the culturally responsive mathematical concepts of the communities observed in a respectful way. Therefore, a blend of ethnographic and phenomenological methods was used to collect and analyse the studied phenomenon (Cresswell, 2013).

A purposive sampling strategy was used to select participants for this study. Creswell and Poth (2016) suggest that such kind of sampling will intentionally identify a group of people that can best offer insight to a phenomenon under investigation. This sampling strategy centered on the perspectives of both *Doko* producers and *Doko* users. We choose to interact with both *Doko* producers and users as these community members likely possessed life-long experiences and knowledge of the traditional customs associated with *Doko*.

In total, eleven community members participated in this study. Two *Doko* weaving experts of the Ahaley community were central sources of information regarding the practices of *Doko* production. In addition, two village members from the Ahaley community and seven village members from the Tamangdara community represented a sample of typical *Doko* users. All participants ranged in age from 50 to 70 years old.

Observations, semi-structured interviews, and content analysis methods were used to collect the data for this study. Information was gathered through actively engaging with participants as they interacted with the *Doko* within their own home contexts. Acquiring data from these multiple lenses allowed us to collect a broad range of perspectives, rather than simply relying one data source.

To inform the design of observational, interview, and content analysis criteria, our team performed a preliminary observational review of *Doko* anatomy. Using two baskets locally available within our college campus, we broadly identified the primary mathematical concepts embedded within the basket's structural components. A *Doko* has two parts: the main body and its cover (Figure 4). Major observations of different mathematical concepts existing in the two primary Doko components were recorded. One smaller sized *Doko* was deconstructed to further identify the prominent geometric patterns of the basket's structure. This period of interim analysis allowed us to build a deeper understanding of the *Doko's* structure, and ultimately served to guide our subsequent data collection procedures (Huberman & Miles, 2002; Johnson & Christensen, 2019).

Figure 4: A traditional Doko bamboo basket (right) and its detachable cover (left) that were used for interim analysis.

Following this preliminary session, our team observed a demonstration of traditional *Doko* weaving led by a local craftsman from the Ahaley community. The basket maker explicitly demonstrated each step of the *Doko* creation process including the gathering of the raw materials, weaving the basket's body and cover components, and creating a finished project. Detailed field notes were recorded throughout the entire process to document the observational data. To further understand these concepts, our team participated in a some of the weaving steps. Engaging with the expert basket maker allowed for opportunity to ask questions and review their perspective on both the cultural and mathematical significance of the *Doko*. Such organic conversations provided an open context for a more formalized interview process.

After completion of the demonstration and observation process, we conducted a series of semi-structured interviews across both communities. The previously engaged basket weaver was joined by a secondary elder craftsman. These two weavers participated in a joint interview with our research team. Interview items focused on *Doko* design and creation, alongside perceptions of the ecological, economic, and cultural aspects generally related to the baskets. Following the interviews with the *Doko* weavers, we conducted two semi-structured interviews with groups of basket users from each village community. These sessions featured questions regarding the use a *Doko* and its general durability, personal attachment of people to the use of *Doko*, and local historical perspectives regarding the tradition and use of the items. The face-to-face and semi structured nature of these interviews provided opportunities for flexibility in interview administration. The approach created an adaptable environment which allowed for a holistic exploration of the overarching place-based themes central to this ethnomathematics study.

A concept-driven approach was followed to analyse the themes directly from the observation and interview data. The responses from the interview were first transcribed as the original data source was an audio recording. All field notes were compiled and expanded into a word processing file. Data analysis was initiated by first segmenting the data into meaningful analytical units. An inductive coding process was used to mark the segments of data with symbols, descriptive words or category names. The codes were then analysed for relevance against the overarching a-priori place-based themes of ecological, cultural, and economic significance. The final results were condensed in content, narrowing down the information to write into a final coherent narrative.

At the time of the community interviews, one sample *Doko* was collected from the Ahaley study site. Using ethnographic observation techniques (Whitehead, 2005) and following a visual content analysis approach (Johnson & Christensen, 2019), a detailed anatomical investigation of this *Doko* was performed. The main body and the cover of the basket were segregated. Each major dimension was measured using a standard metric measuring tape. Angles of woven intersections and structural corners were measured using a handheld protractor. Through this process, we were able to record standardized measurements for the geometric patterns observed in the preliminary study. Importantly, these concepts further confirmed and expanded upon through follow-up communication with the expert basket weavers. The physical Doko served as non-reactive object for detailed observations, thus providing a stable source of information to analyze alongside data collected orally (Bowen, 2009).

Triangulation of the multiple data sources was used to strengthen the validity of this study. Interview data collected from both *Doko* producers and users was examined independently and then holistically to corroborate evidence from different sources in regards to the deductive themes (Creswell, 2009). Follow-up interviews were also conducted with participants to ensure the accuracy of their perspectives and statements. Following suggestions from Gibbs (2007), additional procedures were used to evaluate the reliability of findings. Reviews of transcripts and codes were conducted by the research team, and inter-coder reliability was assessed to compare agreement between coded qualitative data.

Collectively, the thematically analyzed data and the documentary analysis of the physical *Doko* outlined the preliminary ethnomathematical concepts relating to these traditional baskets. A comprehensive review of these findings allowed for further consideration on their potential to be used as educational assets. Reflection on these opportunities within the broader lens of a place-based pedogeological approach are further described in the subsequent sections.

Results

This section presents findings of content analysis and participant interviews concerning the ethnomathematics concepts of *Doko*. Upon detailed inspection, numerous mathematical concepts were found embedded within the anatomy of a *Doko*. These include patterns within woven geometry of the basket's sides and also the geometry of a *Doko's* overall structure and morphology. Mathematical models that can describe these concepts are subsequently presented.

Interviews with local individuals revealed how the weaving and use of *Doko* are intricately connected to the lives of community members across the place-based educational themes of ecology, culture, and economy. The documented mathematical concepts and socio-cultural significance of these baskets could present powerful opportunities for place-based ethnomathematics education in Bhutan.

Mathematical Concepts – Woven Patterns

The base of a *Doko* has only two dimensions. The most common weaving technique results in a basket with 4 x 3-dimensional relationship, where `4` and `3` represents the number of quadrilaterals of convenient shape (square/rectangle) and size of the *Doko*. Similarly, another weaving technique can create a *Doko* with a 5 x 4-dimensional relationship. These larger dimensions are less commonly constructed.

The orientation of 4 x 3 results in four rows and three columns of quadrilaterals formed from woven bamboo strips. Each strip is made of a pair of small strips: one hard (green) and another soft (white) bamboo strips (Figure 5).

Figure 5: Image of the base of a *Doko* with bamboo strips of alternating colour.

Two pieces of thick and strong bamboo strips are fixed diagonally in order to make the base strong and durable. These detachable basal strips establish the patterns for weaving the *Doko*. It has been noted that the length of the diagonals also determines the size of the *Doko*. That means larger the length of the diagonals, larger the overall size of the *Doko* once completed.

A patterned arithmetic progression can be found within the woven structure of the *Doko's* walls. This pattern changes along the vertical dimensions of this basket. The change in pattern corresponds with the change in shape of the *Doko's* body. The first line of the geometric pattern is formed via the combination of small equilateral triangles, quadrilaterals, pentagons and hexagons. Notably, the presence of pentagon is observed only at this location within a *Doko`s* structure.

An angle developer (Ad) is a single bamboo strap which develops six small equilateral triangles (whose bases are adjacent to each other) in such a way that an interstitial hexagon is created. Each Ad has two branches. An Ad is inserted at every edge of the *Doko's* four sides. Each Ad system results in a patterned combination of small equilateral triangles and hexagons stemming from the original angle.

Ad angles were measured from the sample *Doko* provided by the community members. Table 1 records the change in the Ad measured across the progressing horizontal rows that make up the structure of this donated *Doko*. The right most column records the average Ad for each horizontal row.

Row No	**Angle (in degree)**				**Average**
R1	70	60	70	70	67.50
R2	60	73	70	70	68.25
R3	60	63	70	63	64.00
R4	60	63	63	60	61.50

Table 1: Angles of Angle Develop (Ad): Each row has four angular developers (Ad) fixed. Here, R1 is the first row (from the base) from where the insertion of angle developer starts. R4 is the last row from where the closer of developing the Doko`s structure starts.

The measured angles demonstrate inconsistencies that deviate from symmetrical *Doko* design. Under normal conditions, a traditional woven *Doko* should have 70° angles for all basal Ad's within the first row. However, one 60°Ad was observed within the basal row of this measured *Doko*. The differences in the angles of the first-row result in progressing Ad's that are not equal. Accordingly, the angle of Ad increases from 67.50° to 68.25° from the first to the second row. This indicates that body of the *Doko* expands in both by vertical and horizontal dimensions. Subsequently, the Ad in rows three and four continues to decrease. This pattern is expressed with a decrease in the horizontal dimension as the basket structure rises vertically.

Geometrically, the woven bamboo structure of the *Doko* follows a tessellated pattern for the entirety of the basket. This tessellation takes the form of small equilateral triangles and hexagons (Figure 6). The Tessellation originates in the first row of the Doko and terminates at the baskets rim with a final line of hexagons.

Figure 6: Image of two separate *Doko* displaying their side faces with hexagonal patterns.

Hexagons are a prominent and observable shape within the tessellated *Doko* walls (Figure 5). Insertion of the Ad will ultimately dictate how these hexagons are expressed. During construction, the basket weaver will craft six equilateral triangles in a circular manner. The bases of these triangles are placed adjacent to each other. Once complete, a hexagonal shape emerges in the centre of the triangles.

Noticeably, these hexagons are not evenly distributed across the sides of a basket. The shape of hexagons depends up on the size of the *Doko*. As the Ads are inserted in an alternative manner, a spiral of hexagons begins to emerge along the side of the basket. The number of hexagons present within each horizontal row corresponds to its sequence in this spiral progression. Table 2 displays this recorded sequence.

Interestingly, the presence of hexagons on each side of the basket follows a pattern of its own. As depicted in Table 2, the numbered pattern for hexagon placement along the 'small side' of a *Doko* is as follows: 4; 5,5; 6,6; 7,7; 8,8. This pattern shows that that there are 4 hexagons between the base of the *Doko* and the first horizontal row. Progressing upwards along the wall, the first Ads are inserted. The insertion of the Ad's results in an increase to 5 hexagons. Within the third row no, Ad is inserted, and the pattern of 5 hexagons is maintained. When Ads are again inserted into the fourth row, another two rows of 6 hexagons are developed. Accordingly, by the ninth row another increase of two hexagons can observed. This pattern is replicated on the other small side of the *Doko* as the spiral pattern is present on all sides, the largest side of the basket completes its course with the highest amount of hexagons in its terminal row. This asymmetrical spiralled pattern is maintained and observable on various *Doko* regardless of their original dimensions.

	On Spiral move (Hexagons)			
Spiral Row	Side1 (LS)	Side2 (SS)	Side3 (LS)	Side4 (SS)
1	4	4	5	4
	4	4	6	5
2	5	5	6	5
	5	5	7	6
3	6	6	7	6
	6	6	8	7
4	7	7	8	7
	7	7	9	8
5	8	8	9	8
	8	8	9	

Table 2: Developmental pattern of hexagons.

Mathematical Concepts – Structure & Morphology:

The overall morphology of a *Doko* can be described as a truncated pyramid. Generally, the body of a *Doko* has two small sides and two large sides. The base forms another smaller face. The surface area of each of the baskets faces and total surface area of entire shape and the volume of the *Doko* can both be calculated.

The base of a *Doko* is either in rectangular or in square shape. The walls of the basket are roughly trapezoids. All surface areas and volume of *Doko* can be calculated using following formulas:

Area of Trapezoid: The area, A, of a trapezoid is:

$A = \frac{1}{2}h(b_1 + b_2)$ where h is the height and b_1 and b_2 are the base lengths.

Volume of truncated pyramid: $V = 1/3 \times h \times (a^2 + b^2 + ab)$ where "V", "h", "a" and "b" are volume of the truncated pyramid, height of the truncated pyramid, the side length of the base of the whole pyramid, and the side length of the base of the smaller pyramid. (Learn, 2021). The volume of a truncated pyramid = Volume of the whole pyramid - Volume of the small pyramid.

Mathematical Models to Describe Doko Construction

Mathematics model 1:

Formal knowledge of Arithmetic Progression (AP) was not described by the expert weavers during the face-to-face interviews. However, the complex *Doko* structure demonstrates that these weavers are applying the model in basket crafting. The following model of arithmetic progression was recognized within the structure of the *Doko*:

AP: T_n= a + (n-1) d where a= first term; d= common difference; n= no.of terms. This model is taught currently in Bhutanese schools. It is illustrated in the current syllabus of class XI Mathematics, Unit 1, Chapter 1: Sequence and Series (O.P. Malhatra, 2010)

Mathematical model 2

Formation of quadrilaterals follows the number pattern: (5 x 4) bamboo strips forming (5-1) (4-1) = 4 x 3 =12 quadrilaterals (*Figure 7*). Five pairs of strips make 4 rows and 4 pairs of strips make 3 rows of the quadrilaterals. This is the base of a normal sized *Doko*. Similarly, (6 x 5) bamboo strips forming (6-1) (5-1) = 5 x 4 =20 quadrilaterals. This is the base of a large sized *Doko*. From this observation following *Mathematical model 2* has been understood. If the base of Doko has been started with 5 bamboo strips for the length and 4 bamboo strips for breadth, then the no.of quadrilaterals we need to consider is always product of (5-1) and (4-1). This can be generalized as *Number of quadrilaterals= (L-1) (b-1)* where L = no.of bamboo strips used to establish length , and b= no.of bamboo strips used to establish the breadth of base of a *Doko*. According to the *Doko* expert 2, a *Doko* has only two dimensions of its base either (4x3) or (5x4). *Dokos* with larger base are used in rare cases where as the one with small base are used commonly, and larger the dimension of base, larger the size of *Doko* is.

Ecological significance of *Doko*

Both *Doko* producers and users shared that successful basket construction is tied to the ecological availability of a specific bamboo species (Figure 7a). This variety, *Dendroncalamus hamiltonii*, is noted for its structural characteristics of elasticity and durability. These favourable attributes support the weaving of strong baskets. In the local dialect, this bamboo species is referred to as *Choya bans (*चोया बाँस in Nepali). Literally translated, *bans* refer to bamboo, and *Choya* refers to the strips used in basket weaving.

The favourability *Choya bans (*चोया बाँस) as a basket weaving material has been observationally tied to the plant's age. Noting this topic, one expert weaver shared the following information:

> *Among all varieties of bamboos, 8 to 9 months old Chayo bans is best for weaving Doko. There is one more special type of Bamboo called as "Neeva* [नीभा in Nepali]*" which is also used to make Doko , but it does not last long. Neeva is available only at high altitude approximately 2000m above the altitude of Samtse.*

Figure 7: Images captured during a live *Doko* weaving demonstration with an elder basket weaver from the Alhaley village: (**a**) preparing the *Choya bans(*चोया बाँस) bamboo into strips; (**b**) beginning the baskets base; (**c**) building the basket's walls; and (**d**) finalizing the basket's rim.

Following this description, a secondary weaving expert shared:

> *If the choya bans is older than 10 months then it is not good for extracting choya (strip) for making Doko. The Doko will not last long. However, if the bamboo`s top is broken then choya (*चोया*) can be extracted even after 10 months, and the Doko lasts long.*

The interviewed *Doko* weavers further added that *Choya bans (*चोया बाँस) typically grows in areas dominated by rocky cliffs. This topographic association was attributed to locations where consistent sunlight is present. The interviewees shared that bamboo grown within these conditions are often found to be of good quality. The basket weavers described how numerous small tree species traditionally used for animal fodder are typically found growing in these localities in association with *Choya bans (*चोया बाँस). These include plants such *as Khaniu (*खनिएउ)*, Maina, Lampatay* (लाम पाते)*, Pani sas* (पानि साज), in local dialect.

Multiple weavers and basket users shared that *Doko* constructed of *Choya bans* will last longer than baskets made out of different bamboo species Accounts varied from the basket lasting between six months to two years depending on its quality and frequency of use. Fire hardening techniques are often used to strengthen the basket's integrity. One basket weaver described how newly woven *Doko* can be hung over a smouldering oven for two to three days. Such practices help dry the basket properly, and the resulting the *Doko* can last for up to two years or longer.

Cultural Significance of *Doko*

Narratives from interviews revealed that the production and use of *Doko* is deeply tied with the cultural practices of the Samtse communities. One weaving expert described how the baskets have a deep cultural connection to the lived experiences of the village inhabitants. Accordingly, people within the village value *Doko* as an important cultural artifact.

Community members interviewed shared that knowledge of weaving *Doko* is usually limited to a few individuals. Expertise of this traditional craft was formerly widespread within village communities, however the population of skilled basket weavers within both study sites is now reportedly declining. Production of these baskets and knowledge to create them rests primarily in a core community of skilled village elders.

Conversations with the elder basket weavers share how harvesting practices are dictated by longstanding cultural tradition. Harvesting follows an avoidance of specific days that are deemed culturally inappropriate for natural material collection. These days, described as *Saran (*सरन in Nepali) in the local dialect, mark the times where the bamboo is not allowed to be cut. Local custom dictates that crafting products with bamboo harvested during of these traditional times will result in items with poor structural integrity. One basket weaver described this belief in detail:

> *The day when there is "Saran (सरन)" in the calendar is a bad day for cutting bamboos and any other woods. If cut, then these materials will decay very soon and the products [crafted from this bamboo] will not last long. However, some people in the village do not believe this.*

Another basket weaver further elaborated that this belief was inherited from their ancestors. Observance of this belief by the interviewed weaving experts indicates that there is culturally continuity within these communities. However, the weavers did share that these beliefs are not universally upheld by all village residents.

Once the basket is crafted, the *Doko* can have many uses, both culturally and practically significant. Local communities use the baskets during traditional rituals to aid in carrying and serving of customary foods. During marriage ceremonies, an event known as a grand puja, or large communal prayer recitations, are a common occurrence. In addition, annually occurring cleansing rituals, known *Moenlam Chhenmo (Purans- पुरन in Nepali),* often feature *Doko* are used to store, carry and serve foods and drinks. As such practices are common for all community members, it is customary for every household to possess at least one or two *Doko*.

Interviewed participants revealed that outside of cultural events, these baskets are mainly used for carrying heavy materials. Typical items that are carried within a *Doko* might include firewood, fodder grass, stones, cow dung, maize, yams, paddy hay, or millet saplings. Commenting on the versatility of these baskets, one interviewee remarked that "no other items have as many uses as that of *Doko*." The baskets may also be used for a variety of agricultural tasks. Some examples include turning the basket upside down as temporary shed for newly hatched chickens to protect them from wild animals at night and as a standardized measurement vessel for farm products during the time of harvest.

The majority of the interviewees shared that common village tasks would be difficult to complete without *Doko*. The significance of the basket as a symbolic item within cultural and tradition practices was consistently described an integral characteristic leading to the *Doko's* perceived importance. Likely resultant of such common use, multiple interviewees shared beliefs regarding the contents of a *Doko* are prevalent within their communities. One participant elaborated on this topic that, "some people have a belief that seeing an empty *Doko* brings a bad omen" or bad luck. However, some interview participants shared that while such beliefs are acknowledged, they are not universally held by all community members.

Economic Significance of *Doko*

Both weavers and community users described that while *Doko* are valued items within their villages, crafting of these products is not completed on a for-profit basis. Instead, baskets are traditionally woven and sold within the village for local use. *Doko* weavers create these baskets simply to meet the demand of the local user community, and do not produce excess for additional sale outside of the villages.

The majority of the basket users shared that *Doko* have been used (at least) since the time of their grandparents, if not longer. One villager in their seventies shared that she had been using *Doko* since she was a child. This person further recalled how the transactions of baskets had changed over time:

> *During my childhood there was no trend of buying or selling of Doko…[instead] there was a barter system of trading [one basket for a] full days labour in the field. Yet, if we had to buy the then cost of a Doko a lot less than it is today.*

Interviewed expert basket weavers described how the price of a *Doko* can vary depending upon its quality and use. *Doko* that are woven specifically for carrying water, known as *Pani Doko (पानि डोको in Nepali)*, cost less than a standard basket that can be used for multiple purposes. Specialty baskets, known as *Dhakar (ढाकर) Doko*, are often sold at the highest price.

As crafting is dependent on material availability and local demand, *Doko* weaving occurs as seasonal work. The process of weaving an entire basket can take anywhere from five to six hours for completion. The use of material is rather efficient for crafting these baskets. One expert revealed that a single "bamboo [shoot] with its good length is enough to complete a *Doko*."

Discussion and Conclusion

Through a holistic review, this study demonstrated how ethnomathematical concepts are woven into the traditional *Doko* bamboo baskets common to the cultures of southern Bhutan. Ethnographic observations and a visual content analysis approach revealed geometric patterns present within basket's sides and also the geometry of a *Doko's* overall structure and morphology.

Furthermore, interviews with both basket weavers and basket users demonstrated how the *Doko* are intricately connected to the lives of community members across the place-based educational themes of ecology, culture, and economy. When considered in context with the mathematical concepts present in the basket's anatomy, this thematic content could offer promising opportunity for facilitating place-based mathematics education in Bhutan.

Areas of alignment within the current mathematics curriculum may present the most relevant scope for the integration of ethnomathematical potential of *Doko*. Geometric applications offer a prime example for such integration. Tangrams (dissection puzzles featuring simple geometric shapes) are typically used to teach basic geometric relationships within Bhutanese schools. Often abstract, the use of these traditional tools situated in lecture-based teaching approaches can lead to perceived student disengagement (Dolma, 2017). Incorporation of locally sourced *Doko* and its simple to complex geometric patterns provide context for such a lesson. Use of these *Doko* would be a step towards creating a more relevant and place-based learning experience.

In addition to geometric relationships, this study has revealed multiple avenues where the *Doko* could be used as an ethnomathematical asset for teaching and learning. The baskets' overall structure and morphology could be used as teaching aid for exploring concepts such as trigonometry, volumetric calculations of truncated pyramids, area calculations of trapezoids, and mathematic models on Arithmetic Progression (AP). The richness and versatility for which these baskets could be used would truly be limited by the imagination and efficacy of the educator. Using these baskets as a visual aid alone, however, represents only a narrow portion of their ethnomathematics potential. A deeper educational contribution could be found if teachers and students sought to explore these items through an interdisciplinary lens.

Bridging mathematical concept lessons with discussions on the ecological, cultural, and economic significance of these pieces of material culture could work to further deepen students' understanding of both mathematic and integrated concepts. Such explorations would be well situated through the implementation of a community as classroom approach (Vander Ark et al., 2020). On one hand, classroom invitations could be extended to local community experts and parents to share experiential knowledge in regards to *Doko*. On the other hand, instructors could work with local communities to organize field trips for their classes to study *Doko* in their home settings. Such learning journeys would revolve around taking students to the nearby rural village community where the people weave and use *Doko* regularly. These explorations could be structured around inquiry-based investigations, with student led explorations of the basket's significance in local economic, ecologic and cultural practices. Through student-centered interviews, live demonstrations, and hands on design-thinking and authentic construction activities, students could engage with the community and content, applying their developing mathematical understanding in real time.

Incorporation of such community-based learning partnerships could lead to improved outcomes in engagement and achievement (Civil, 2010; Sheldon & Epstein, 2005). While challenges will likely exist for the implementation of such ethnomathematics techniques (Sunzuma & Maharaj, 2019), the potential alignment with the holistic goals of Educating for GNH shows promise. Research on the efficacy of these implementation pathways would be critical next steps in studying the educational capacity of *Doko.* Ultimately, educational experiences that embrace this place-based approach of ethnomathematics could concurrently work to preserve local culture while also providing experience for conceptual mathematical learning.

The application of ethnomathematics practices as described would present a strong foundation for the nation's most recent and emerging educational reform effort: the Bhutan Baccalaureate program. The Bhutan Baccalaureate is rooted in the belief that building a strong, secure and prosperous world has to begin with realizing and developing dispositions towards service for all communities. As the perceived next step beyond the Educating for GNH Initiative, the Bhutan Baccalaureate strives to create the right environment for the development of future leaders. These conditions emphasize building confidence and self-efficacy to approach the multidimensional challenges and opportunities of the modern era. In short, Bhutan Baccalaureate seeks to equip students with the strategies to build a more just and harmonious society in both local and global scales.

At its foundation, the Bhutan Baccalaureate has a distinctive understanding of the individual and their place in society. Within this approach, academic (cerebral), emotional, physical, social, and spiritual dimensions are all recognized as essential targets for educational development (Kapur, 2021). Every learning experience of the Bhutan Baccalaureate is geared towards the development of these five areas. Environments which enable learners to engage cerebrally should likewise provide opportunity to grow in the other dimensions.

Following this model, ethnomathematics offers an approach that allows students to learn across dimensions. Incorporation of place-based concepts alongside traditional academic mathematics offers opportunities for learners to build understanding across local and global communities. The study of *Doko* as an educational asset would offer one tangible track towards the implementation of these new educational visions of Bhutan.

If ethnomathematics were to expand in Bhutan, a suite of other cultural concepts could offer interesting directions for continued research. These concepts could include textile weavings, drawings, paintings and rituals that are being practised in the diversity of Bhutanese culture, traditions and festivals. Embedded mathematical concepts within these practices span a range of discrete mathematics (mostly on counting), geometry (on shape and patterns), trigonometry (angles), permutation (arrangement of patterns and colour) and combination (possible ways of selecting the choices). Explorations into traditional or shamanistic healing practices (which are largely guided by concepts of probability) could also offer potential for ethnomathematics investigations. Ultimately, through centering both student and local experience into mathematics education, Bhutanese educators could begin to align their practice to the modern reform efforts of the nation's educational vision.

Bamboo *Doko* presents a tangible piece of material culture that has rich potential for teaching and learning through an ethnomathematics approach. We hope that this synthesis can offer an entry point for additional research and applications of the deep possibilities of ethnomathematics study in Bhutan. As a unified nation with a diverse span of identities, a growing tradition of ethnomathematics research could be a solid step towards the preservation and promotion of culture.

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