Art & Mathematics: A Brazilian TV Series

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Dedicated to the memory of Professor Ubiratan D'Ambrosio (1932 - 2021)

Abstract

Art & Mathematics is the overall title for a Brazilian series of TV programs on mathematics produced by TV Cultura with the support of the Ministry of Education in 2001. Its aim was to entertain while discussing everyday life and the role played in it by mathematics and art. The intention was to bring onto TV screens unusual, interdisciplinary, inspiring and fascinating examples that would broaden viewers' perceptions of both mathematics and art. Logic, symmetry, perspective, chaos, fractals, poetry, music and mathematics were discussed and connected with interesting illustrations, some of them based on Brazilian artistic and cultural aspects.

Keywords: Art; Mathematics; Recreational; History; Education; Culture

Introduction

On November 14, 2001 the Brazilian government television aired the *Art & Mathematics* series at prime time showing 13 episodes, one per week, lasting 25 minutes each, reaching an audience of almost a million (Nascimento and Barco, 2007). The purpose of this series was first to entertain, and second to educate, especially children and adolescents. The authors, hired as consultants (Barco was also hired as a presenter), were inspired by a wish to shift the public perception of mathematics beyond the technical, taking into account a humanistic and historical approach. Although learning was the secondary purpose, it is important to note that most of the mathematics presented in all the episodes dealt with intricate concepts at a basic level. Some of them were related to number patterns, geometry and perspective, topology, fractals, chaos, derivative and logic, among others.

For most people the mere mention of the word mathematics conjures up memories of complicated rules. However, these rules are merely tools. A way to bring classical as well as new mathematical content to a large audience was to make a TV series on the subject. Of course, such a production was very difficult because it is not simple to translate mathematical concepts into simple terms in a few minutes for a large audience and keep their attention. An interdisciplinary team involving artists, screenwriters, musicians, poets, mathematicians, physicists, directors and art historians would help in the task. Some past and recent inspiring examples of such series elsewhere exist, such as "*The Story of Maths*" (BBC 4, 2008) or "*The Code*" (BBC 2, 2011) both hosted by the British mathematician and popular science writer Marcus Peter Francis du Sautoy (*b.* 1965). An inspiring documentary series was "*Life By The Numbers*" (WQED Pittsburgh / PBS, 1998), which had the British mathematician and popular science writer Keith

James Devlin (*b*. 1947) as a consultant (Devlin, 1998) and was hosted by the American actor, film director and political activist Daniel Lebern Glover (*b*. 1946).

Learning could be viewed as an artistic process. Both mathematicians and artists practice their activities in many ways, but a common one is *repetition*. This procedure is one of the simplest ways to gain knowledge. As a prolific author on the subject, Hans Freudenthal (1905 - 1990), a German-born Dutch mathematician, wrote on mathematical education (Freudenthal 1973, 1978). He believed that mathematics is a human activity, which means that it is a process. For example, he stated that a (textbook) mathematical problem usually describes "the nucleus of a situation in too abstract a manner". As a mathematics teacher and researcher at the university, he often used the approach of the Bourbaki group, using set language to precisely introduce theorems to mathematicians. Freudenthal observed the need to merge "everyday reality in secondary education mathematics as a source of learning, not just for applications, emphasizing the relevance of richer thematic contexts" (Freudenthal, 1973).

Like artists, mathematicians take their inspiration from a surprising range of sources. To be clear, the *Art & Mathematics* series made use of similarities between these two human activities, believing that cognitive development (at least in math) usually starts from the concrete to the abstract, following Freudenthal's proposals (Freudenthal, 1973). At a higher level, even abstract artworks could be compared to mathematical ideas. However, the illustrations used in this series were kept as concrete and simple as possible.

As everybody else would expect, adjectives such as boring, dull, irrelevant, lifeless, rigid and tedious would not be applied to math. Far from being a dry and monotonous subject, mathematics is a powerful tool that handles not only nature but also human aspects, such as art. One aim of this series was to show that mathematics is beautiful: exquisitely austere, intellectually elegant and beautiful. Yes, it's beautiful to look at. Even when some concepts are very abstract, they could be compared to art. This view is not new: the British philosopher, mathematician, logician, writer, historian, political activist, social critic and Nobel laureate Bertrand Arthur William Russell (1872 - 1970) once said that mathematics has "a supreme beauty capable of a stern perfection such as only the greatest art can show" (Russell, 1917). The American engineer and mathematician Jerry Porter King (*b*. 1935) also wrote that the motivating force for mathematics is *beauty*, followed by its goal: *truth*. Finally, the significance of what mathematical truths reveal: *reality* (King, 1992).

Some mathematicians may think of themselves as artists - which they indeed are - but often artists do not see themselves as mathematicians. The Brazilian painter Wesley Duke Lee (1931 - 2010) in the third episode of this series (*The Artist and The Mathematician*) said: "mathematics is knowledge that has a certain kind of *beauty*. A beauty different from music or painting. Distinctive, but equally *enjoyable*. The beauty of mathematics is the *aesthetics of reasoning*" (Nascimento, 2017).

Another purpose of this series was to show that mathematics should not be taught as isolated topics. One way to present its ideas would be through art, which has an interesting and fuzzy historical evolution, featuring different people in different places, formats, contexts and times.

Authors such as the American mathematician Reuben Laznovsky (*b*. 1927), well-known by his pen name Reuben Hersh, advocate a historical understanding of mathematics (Hersh, 1997). In particular, the present authors justify the use of Brazilian artworks due to Hersh's book (1997), which once affirmed that "*from the viewpoint of philosophy*, mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context" (Hersh, 1997). Universal artworks were also presented in this series, mainly because mathematics is also a worldwide language, but the authors preferred to illustrate math ideas using, as far as possible, art and cultural representations from Brazil. This choice was also based on ethnomathematics (D'Ambrosio, 1985). Another important goal of this series was to present suggestive parallels between art and math without any mathematical or artistic prerequisites.

In this work we present this series. Mathematicians often argue about beautiful theorems and beautiful proofs. Similar to art, issues of beauty, elegance, pleasure, grace, simplicity, style, clarity and practice are intriguing subjects. This is also mathematics.

The Project

Art & Mathematics was produced by the Secretariat of Long-Distance Education ("*Secretaria de Educação a Distância*" – SEED) of the Brazilian Ministry of Education ("*Ministério da Educação* - www.MEC.gov.br), TV School ("*TV Escola*") and the Padre Anchieta Foundation, where TV Cultura is based (www.tvcultura.com.br). Everything started with an interview with one of the authors at the beginning of 2000 about his view on possible connections between art and mathematics for a television series.

After initial strangeness, because as a general rule mathematics is the last topic to which television gravitates, one of the basic questions in this interview was the following: is there anything in common between mathematics and art?

Surprisingly, the answer was *yes*. Art and mathematics permeate nearly every aspect of our lives. Regardless of the type of art - painting, sculpture, music, theater, dance, film or poetry - mathematics and art use abstraction, exercise the imagination, and consider primordial objects, like forms or sounds with numbers. The project should also make use of Brazilian artistic and cultural examples (D'Ambrosio, 1985).

Both art and mathematics use common terms such as aesthetics, perfection, organization and rigor. They seek balance, harmony and simplicity. They are considered *languages*. Anyone can see that they strive to look for patterns, and interestingly, even the absence of a pattern is celebrated.

The invitation came due to the writer's long experience of publishing a monthly column about education and mathematics in a popular Brazilian science magazine named *Superinteressante* (www.superinteressante.com.br), first published in September 1987. This mathematical column was entitled "*Dois Mais Dois*" ("Two plus Two"). From the first issue, it quickly became the most popular magazine on the subject. The other reason was related to a special prize that was won in 1975 by Fundação Padre Anchieta, which aired the Tele-School Project. This was written and presented by Luiz Barco (*b*. 1939). It was designed for Brazilian secondary school students (aged 12 and up) to explain mathematical concepts and operations. The fourteen-episode winner (19 min 11 s) was mentioned as a "programme [that] ingeniously expounded the concept of 'relative whole numbers'" (The Japan Prize, 1975).

The Brazilian actors that hosted *Art & Mathematics* series were Joyce Roma (*b*. 1978) and Edson Cruz (*a.k.a*. Montenegro, 1957 - 2021), who put their drama skills to good use, introducing each episode with an inviting tease and narrating most of what was to follow. In each episode, one of us also introduced some mathematical concepts from a type of math bureau and offered commentaries. Almost no formulas were presented, and intuitive understanding, especially with visual cues, was emphasized throughout.

Table 1 presents the thirteen episodes in order with the main topics (some examples and proposals were not published in the final version). They were entitled: *i*) From Zero to Infinity; *ii*) Art and Numbers; *iii*) Artist and Mathematician; *iv*) Order in Chaos; *v*) Symmetries; *vi*) The Golden Number; *vii*) Music of the Spheres; *viii*) Mathematics of Music; *ix*) Time and Infinity; *x*) Form inside Form; *xi*) Transforming Form; *xii*) Chaos; *xiii*) Beauty.

Table 1

All thirteen episodes of *Art & Mathematics*, including original proposals (not all of which were included in the final version) and links to each video.

#	Episode	Main topics
1	From Zero to Infinity ("*Do Zero ao Infinito*") 26min:41s	A brief resume of this series. *Where do we come from*? *What are we*? *Where are we going*? These are deep questions done by civilization. It is also a painting by French artist Eugène Henri Paul Gauguin (1848 - 1903) elaborated in 1897. In this work he wrote, without question marks or dashes, as a title in the upper left corner: *D'où Venons Nous* *Que Sommes Nous* *Où Allons Nous*. Why do humans create art? Why do we require art? One answer is that art is what makes us humans, a quote attributed to the Armenian orientalist and Director of the Hermitage Museum (in Saint Petersburg, Russia), Mikhail Borisovich Piotrovsky (*b*. 1944). One can change *art* in such questions by another very important knowledge produced by civilization. Why do humans do math? Why do we require math? One answer was given by the Italian polymath Galileo Galilei (1564 - 1642), when he claimed that "the *Book of Nature* is written in mathematical language". Nature's riddles can be answered in an aesthetically pleasing way by math. Mathematics is another human manifestation that organizes rational questioning. It deals with measures and values of the physical world as well as abstracted thinking. In a way, artisans, artists and mathematicians work intuitively. Artists use their artwork to make representations. Mathematicians use logic to create both real and abstract representations. Different points of view? Not at all. https://youtu.be/Khf4qSEJeMU
2	Art and Numbers	Cave paintings by primitive men and women are drawings on cave walls found around the world whose exact purpose is unknown. Because animals were the

	("*Arte e Números*") 24 min:47s	most common subjects, some experts believe that such paintings were a form of communication. It was one of the first ways to establish language using patterns found in nature. From ancient times, nature's patterns were drawn in different ways, as flat figures that formed mosaics. In the Middle Ages, the best representation of space emerged, based on perspective and new geometric techniques. In another place and time, arithmetic promoted the discovery of zero, which means "empty," in India around the IX century. The first calculators (*abaci*) at that time were simple holes in series - or slabs covered with sand - where counting pebbles were distributed. This was followed by the later appearance of the decimal system and the first numerical digits, which were related to the first medieval trades and exchanges. https://youtu.be/KiU9o89haSM
3	Artist and Mathematician ("*O Artista e o Matemático*") 25min:51s	*Mathematics and Art United*. Mathematicians and artists see beyond appearances, from immediate forms, and intuitively note, which is known as reasoning aesthetics. Knowledge division into distinct areas is a recent phenomenon in human history. Cultivated men in the Renaissance knew about arts and sciences: mathematics, botany, physics, chemistry, biology, and philosophy. They did calculations and produced magnificent artworks, as did the great Italian polymath Leonardo di ser Piero da Vinci (1452 - 1519). Impressionist and expressionist painters experimented with new artistic directions in the nineteenth century. More recently, a new aesthetic view was raised in 1958: the post-war *Concrete Poetry* manifesto was published in Brazil. The new Brazilian artists worked with spatial word arrangements and mathematically planned poems. This abstract art was an alternative to describe new visual experiences: emotion in colors, shapes and creative forms, searching for a departure from reality. Some of them are now promoted by computers, that take into account mathematical calculations, as fractal figures.

		https://youtu.be/REmRToHF2Uo
4	Order in Chaos (“*A Ordem no Caos*”) 25 min:51s	Look and learn from nature’s patterns: equalities, regularities and repetitions. But even in complex systems, it is possible to see order related to underlying patterns. The *butterfly effect* describes how a small change can result in large differences. The American mathematician, meteorologist and pioneer of chaos theory, Edward Norton Lorenz (1917 - 2008), established once that “a butterfly flapping its wings in Brazil can cause a hurricane in Texas”. Another great artist, the Dutch Maurits Cornelis Escher (1898 - 1972), elaborated new abstract works based on peculiar observations of nature’s patterns and regularities. His work features mathematical objects and operations, including incredible objects, as well as explorations of infinity, reflections, symmetries and perspective.
		https://youtu.be/dce9y1JBI7I (subtitles in English)
5	Symmetries (“*Simetrias*”) 25 min:52s	Any mirror image shows a reflected object, but reversed. In the Renaissance, the Italian painter and mathematician Piero della Francesca (1415 - 1492) elaborated new geometrical masterpieces, such as the *Baptism of Christ* (*c*. 1450). This notable composition was organized following square and circle symmetries. The square represents Earth, and the circle represents the sky. The Holy Spirit is inside the circle, and Christ precisely divides the composition. Piero had a deep interest in theoretical studies on perspective, and much of his work was later absorbed by the Italian mathematician and Franciscan friar Luca Bartolomeo de Pacioli (1447 - 1517). Pacioli wrote *De Divina Proportione* (*c*. 1509), a work illustrated by his friend Leonardo da Vinci. There are symmetrical proportions in poetry and even in music. For example, consider the *Frère Jacques* (“*Brother John*”) nursery rhyme. Today, symmetry breaking is a useful tool in theoretical physics. Their principles are still used by modern artists to promote strangeness and high abstraction.
		https://youtu.be/ycCMca9M6lw
6	The Golden Number (“*O*	“*All is Number*” was the *motto* of the Pythagorean School, the first and most important mathematicians’ group ever. Since the Pythagoreans, the golden

	Número de Ouro") 25 min:24s	number has been considered the best proportion for all forms, meaning harmony or ideal aesthetics. The angles of such geometric golden figures remain the same, as do the number of sides and their ratio. This is an aesthetic criterion followed by different civilizations Since most remote and primitive civilizations, artisans and artists have intuited and/or rationalized in light of this unique pattern. In nature, trees, flower petals, seashells, or the human body itself present the same divine proportion. The Italian mathematician Leonardo Pisano (*c*. 1170 - *c*. 1250), who is better known by his nickname Fibonacci, found an interesting number series that is related to the golden ratio. Even the piano keyboard presents the first numbers of the Fibonacci series.
		https://youtu.be/5oXr-fItVEQ
7	Music of the Spheres ("*Música nas Esferas*") 24 min:16s	Since its very beginning, music from different cultures, has followed the mathematical principles of sound. For example, since the Pythagoreans, musical scales have been expressed in terms of numerical ratios, particularly including small integers. The musical scale was divided into different ratios around the world. How do we hear? Sound produces waves, air pressure disturbances, and the ear captures such air oscillations. Counting, rhythm, intervals, scales, symbols, harmonies, patterns, overtones, time signatures, pitch, and tone. All composer's notations and all sorts of sounds are connected to mathematics. The German composer and musician Johann Sebastian Bach (1685 - 1750) proposed a particular scale of twelve notes that follows a logarithmic ratio.
		https://youtu.be/BQEnC6prwiE
8	Mathematics of Music ("*A Matemática da Música*") 25 min:44s	*We enjoy listening to music.* Music and math are interconnected: one deals with numbers and forms, and the other with sounds. Both are considered languages and represent patterns. Once, the German polymath Gottfried Wilhelm Leibniz (1646 - 1716) said that "music is a hidden arithmetic exercise of the soul, which does not know that it is counting" ("*musica est exercitium arithmeticae occultum nescientis se numerare animi*"). Pythagoreans used numerical terms to express intervals between notes and derived musical tones from geometrical patterns.

		Music is considered the first scientific experiment ever done, by means of a *monochord*. One type of monochord is still very popular in Brazilian *Candomblé*, an Afro-American religious tradition, as well as in *Capoeira*, a Brazilian martial art. Music and mathematics are expressions of beauty that humanity pursues and continues to build. Musical symmetry can be found in different examples, such as Salsa music or the Brazilian *Bossa Nova* and *Forró* styles.
		https://youtu.be/znL_Fc0HULE
9	Time and Infinity ("*Tempo e Infinito*") 24 min:46s	Cinema is the art that best transmits time sense, being able to prolong actions or compact them. Its mechanics are based on simple mathematical rules, the most common of which is the *frame rate*, the frequency (or ratio) at which consecutive images (or frames) are displayed in a film. This rate was established at 24 frames per second. Infinity is an abstract concept that refers to something that has no bounds or is greater than any natural number. Parallel lines meet at infinity. A *Möbius strip* is a one sided, one boundary endless surface proposed by the German mathematician and astronomer August Ferdinand Möbius (1790 - 1868), by giving one end a half-twist and then joining the ends to form a loop. It is possible to reach infinity in many other ways. Most Brazilians know by heart the meaning of infinity and love due to the "*Sonnet of Fidelity*", written by the Brazilian poet, lyricist, essayist and playwright Marcus Vinicius de Moraes (1913 - 1980). "I'll be able to say to myself of the love (I had): / Be not immortal, because it is flame / But be infinite while it lasts," the final verses say.
		https://youtu.be/rWOv_l06KCs
10	Form inside Form ("*Forma dentro da Forma*") 26 min:17s	*A World of Forms*. According to the ancient Greeks, as described by the great philosophers Socrates (*c*. 470 - 399 BC) and Plato (*c*. 428 – *c*. 348 BC), our world is modeled after Forms (or Ideas) patterns. The remarkable Greek philosopher, mathematician and astronomer Thales of Miletus (*c*. 624 - *c*. 546 BC) measured the height of the Great Pyramids by their shadow, taking the observation at the same time when his own shadow was equal to his height. The Greek aphorism

		“γνῶθι σεαυτὸν”, translated into Latin as "*Nosce te ipsum*" ("Know thyself") is credited to this great mathematician, one of the seven sages of antiquity. The Greek mathematician Euclid of Alexandria (*c*. 325 - *c*. 265 BC), often referred as the “founder of geometry”, cataloged isolated math rules (as those proposed by Thales) and deduced new ones from a small set of axioms, definitions and postulates following a logical structure. In this way, Euclid elaborated the first main textbook for teaching the mathematics of forms - *The Elements*. It is possible to have fun with geometrical elements. One of the oldest and most common puzzles is the Chinese *tangram*, consisting of seven flat figures, called *tans*, that are put together to form shapes. In our time, fractal theory introduced a new geometrical world of forms within forms. More specifically, fractals include the idea of a detailed pattern that repeats itself. This term derives from the Latin *fractus*, which means broken or shattered, and was proposed in 1975 by the Polish-born, French, and American mathematician Benoît Mandelbrot (1924 - 2010). He defined in his seminal book “*The Fractal Geometry of Nature*” (1982) the following: “why is geometry often described as cold and dry? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline or a tree.”
		https://youtu.be/dkN4kWms0yE
11	Transforming Form (“*Forma que se Transforma*”) 24 min:44s	Topology is a branch of geometry that is related to the properties of space that are preserved under continuous deformations, such as bending, stretching, and crumpling but not tearing or gluing. The basis of this new math is related to the discoveries of the Swiss polymath Leonhard Euler (1707 - 1783). Topology explains how a continuous deformation of a mug into a doughnut occurs. Art also plays with the incredible possibilities of forms for articulated objects. For example, in literature, the Spanish artist, professor and writer Julio Plaza González (1938 - 2003) and the Brazilian writer, visual artist and poet Augusto de Campos (*b*. 1931) proposed in 1968 the work *Poemobiles*, or object poems:

		two overlayed large cardboard pages projecting three-dimensional pop-ups that form through an interplay of cuttings and foldings. https://youtu.be/Yl7qPTeDIbI
12	Chaos (“*Caos*”) 25 min:54s	The German-born physicist Albert Einstein (1879 - 1955) believed in an orderly world. Once he affirmed that "God does not play dice". He seemed to have felt that the uncertainties arising from the new discoveries of quantum theory were only provisional. Unpredictable behavior may have rules? Which atom will disintegrate first in a nuclear reaction? There are possible explanations based on mathematics. In particular, *Chaos*, as an interdisciplinary theory, is another branch of mathematics and is focused on the behavior of dynamical systems that are highly sensitive to initial conditions. The American mathematician and meteorologist Edward Norton Lorenz (1917 - 2008) summed it up succinctly as “chaos: when present determines future, but approximate present does not determine future.” https://youtu.be/PRl9ipKQZQc
13	Beauty (“*O Belo*”) 25 min:47s	*Art of Abstraction*. Although humanity has always noticed, admired, commented on, and contemplated beauty, the Greeks were the first to discuss and philosophize about its nature. The Pythagoreans first observed a clear connection between beauty and mathematics. One of the first pieces of mathematical knowledge in human history, the *Pythagorean Theorem*, considered the primary main geometric information, has at least six qualities that can be attributed to mathematics in general: *universality*, *objectivity*, *truthfulness*, *aesthetics*, *resistance* and *applicability*. Concerning aesthetics in math, this particular theorem is described by mathematicians as an especially pleasing method of proof, named *elegant*. However, there are not one but many other ways to prove such an old theorem. To best see more examples of such beauty related to mathematical methods, anyone can choose from any of the 370 simple and elegant proofs of the

Pythagorean theorem, as presented by the American mathematician, professor, engineer, and writer Elisha Scott Loomis (1852 - 1940).

The prolific Hungarian mathematician Paul Erdős (1913 - 1996) once expressed his view on the ineffability of mathematics as: “why are numbers beautiful? It’s like asking why Beethoven’s Ninth Symphony is beautiful. If you don’t see why, someone can’t tell you. I know numbers are beautiful. If they aren’t beautiful, nothing is."

Just as great artists can achieve the goal of knowing their names, the same occurs with those who study and discover mathematics.

https://youtu.be/UHlnhr2DMdk

This series won the *Silver Dragon* prize at the II International Scientific Film Festival in Beijing, China, in 2003 (www.bjiff.com). The prize was awarded for Episode 4: “Order in Chaos” in the group *Programs for Youth*. This was not the first prize the series won. In 2001, it was awarded the *Maeda Prize* in the category Education for Youth, during the 28th Japan International Education Prize competition, considering the same episode (The Japan Prize, 2001).

Episode Samples

We briefly present some episodes of the series in more detail below.

Logic, Literature and Movies (Episode 1)

There is a famous passage in *Alice’s Adventures in Wonderland*, written by the English writer, logician, mathematician, Anglican deacon and photographer Charles Lutwidge Dodgson (1832 - 1898), under the pen name of Lewis Carroll (Carroll, 1865):

> “*Alice*: Would you tell me, please, which way I ought to go from here?
>
> *The Cheshire Cat*: That depends a good deal on where you want to get to.
>
> *Alice*: I don’t much care where.
>
> *The Cheshire Cat*: Then it doesn’t much matter which way you go.”

This answer is absolutely logical, in a strict mathematical sense. It is an example of the hidden mathematics behind a very popular book, in which mathematical ideas are simple, gentle, interesting, curious and accessible.

There are a few examples like this – another good recreational book was published by the Brazilian mathematics professor, engineer, storyteller, writer and educator Julio Cesar de Mello e Souza (1895 - 1974). Under the pseudonym Ali Iezid Izz-Edim Ibn Salim Hank Malba Tahan, he wrote an incredible book, *The Man Who Counted*, first published in 1938 (Tahan, 1993; Nascimento & Barco, 2016). It was a worldwide bestseller, with interesting riddles to solve, such as an inheritance quarrel between three brothers in the middle of the desert. Their father had died, leaving thirty-five camels, half of which belonged to his eldest son (17.5 camels), one-third (11.666... camels) to the middle son, and one-ninth (3.888... camels) to the youngest. To solve the brothers' dilemma, the Persian mathematician Beremiz Samir convinces the voyager's friend (not without complaint) to donate his only *jamal* (a camel) to the dead man's estate. Then, with 36 camels, Beremiz gives 18, 12, and 4 jamals to the three heirs, which means that they all profit from the new share. Of the remaining two animals, one was given back to the friend, and the last was claimed by Beremiz as his reward for finding a solution (Tahan, 1993).

Nascimento and Barco (2016) generalized this classical problem of camel division to any number of brothers. They assumed that there were k brothers and that the i^{th} brother desired some fraction $1/t_i$ of the camels, where t_i is a positive integer. They found 22 possible solutions to Tahan's equation, in honor of Mello e Souza's pseudonym.

Another interesting story presented in this episode relates to one of the first movie sessions: *Tom, Tom, the Piper's Son* (1905), based loosely on the well-known nursery rhyme (Bitzer, 1905). In the first scene of this ≈ 8 minute black and white silent comedy with no editing, cuts or even zooms, a crowd is watching a circus show when the thief Tom steals a pig and runs off. He tries to hide when a joker is juggling, but he is chased by a boy and then by the crowd, and the pig also proves difficult to control. The whole scene seems confusing at first sight; it was made in a long shot, similar to a theater presentation with a dozen actors. The story goes on with a few cuts and some hilarious situations until Tom's capture. The difficulty in the first scene arose mainly because the audience was observing a new language – the cinema. In some theaters, there was an explainer to present this new art. It was necessary to give the viewers explanations of what was going on with Tom, the thief, the crowd and the stolen pig.

This is not so different from how to explain new mathematics to students, and this example can be used as another comparison between art and math. Indeed, students often ask how mathematics works, expecting illustrative examples, and sometimes art can help, like the explainer in the first silent movies. On the other hand, mathematics teachers also have doubts, expressed in a question posed by the Norwegian mathematician Torgeir Onstad (*c*. 1945) in his interesting work: "Is the Mathematics We See the Mathematics They Do?" (Onstad, 2017). Mathematics is certainly universal, but does it make sense to claim that two students have identical mathematical concepts? Ethnomathematics, as well as artistic points of view, can help showing that mathematics is plural. Perhaps one of the simplest examples is that there are 370 ways to demonstrate the Pythagorean theorem (Nascimento, 2018). It is also possible to affirm that the simplicity/complexity of art illustrations could enhance motivation, appreciation and understanding of mathematics.

Derivatives of an Image (Episode 3)

Waldemar Cordeiro (1925 - 1973), an Italo-Brazilian designer, illustrator, journalist and art critic, was one of the international pioneers of computer art, first in Brazil. As a member of the Brazilian avant-garde scene that developed Concrete Art in the 1950s, he started a new form of art done by computers in 1968 with the Italo-Brazilian engineer and physicist Giorgio Moscati (*b*. 1934). According to Moscati (Velho, 1993), they had many discussions about art in which computers played a role in production or artwork display. At that time, many artists were tentatively beginning to explore emerging computing technology for use as a creative tool (Noll, 1994).

Cordeiro and Moscati discussed ways to transform a particular image using computers and decided on the derivative concept. Cordeiro chose a particular image of strong and emotive human content: a St. Valentine's Day card (Figure 1*a*). In this original image, a young couple expressed mutual affection through physical contact. This picture should be transformed by a cold, predictable and calculating machine (Velho, 1993). They digitized the Valentine's Day card and wrote a derivative program for image processing (Figure 1*b*). They fixed the image as points in a 98 × 112 matrix (corresponding to 10,976 dots) with gray levels divided by seven, from zero (white) to six (black) for each point, following an arbitrary scale. Basically, if in a row one has the following succession of points from left to right, the "derivative" line has the following structure (Velho, 1993):

Digitized:

0	**0**	**0**	**0**	**0**	**0**	**6**	**6**	**6**	**6**	**6**	**5**	**5**	**5**	**3**	**2**	**1**	**1**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**

Derivative:

0	**0**	**0**	**0**	**0**	**0**	**6**	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**2**	**1**	**1**	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**0**	**0**
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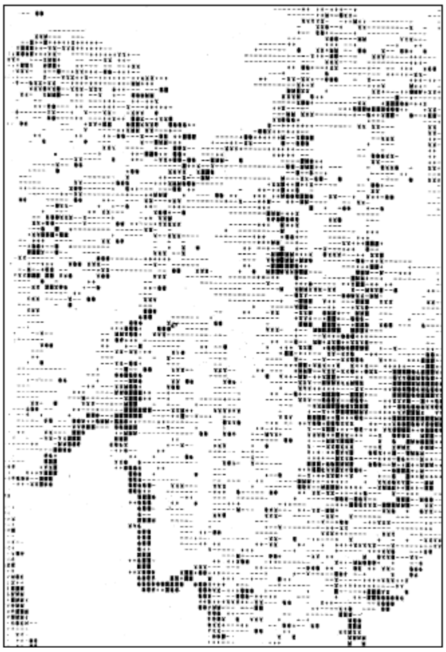
The rule is simple: considering two digits *n* and *m* in sequence, if they are equal, the difference is zero. If not, their difference is taken into account as a positive value only. In the example above, when one has 0 and 6, then 6; if 6 and 6, then 0; if 5 and 3, then 2; if 3 and 2, then 1, and so on.

Figure 1

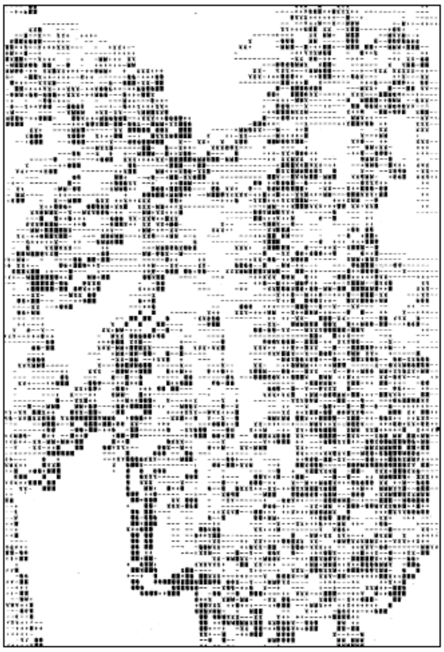
a) A St. Valentine's Day card chosen by Cordeiro for the first visual computer artwork made in Brazil (1969). *b*) Degree zero, or simply the digitized image, with 10,976 points, with each point related to seven levels, from zero to six. *c*) Degree one, the first derivative. *d*) Degree two, the second derivative.

a)

b)



c)



d)

Thus, it is simple to observe the application of the derivative concept: from left to right and point by point. If there is no variation between two points, the result is zero; also, the gray scale helps to visualize the variation between two neighbors. At the left border, Moscati decided to repeat the same values from the previous image (in our example, there is an underlined zero). The result was clear: the computer transformed a shaded image into a contour image, according to Figure 1*c*.

In brief, this mathematical procedure is due to the fact that wherever the intensity of the digitized picture remains constant, the derivative will be zero, and therefore, white; and wherever the intensity changes abruptly from light to dark, the derivative image varies smoothly (Velho, 1993). In this way, it was possible to create the first computer-generated drawing made in Brazil in 1969.

Cordeiro and Moscati solved technical problems related to distortion (plots were printed as 47×34.5 cm^2), among others. As might be expected, the derivative image was similar to the digitized, because they only transformed the image by means of several degrees of light / dark (Velho, 1993). Successive derivatives applied to the same object gradually lost information (Figure 1*d*). Cordeiro and Moscati decided that their work would consist of four images: the digitized and three successive derivatives, which they named “Image Derivatives” (“*Derivadas de uma Imagem*” in Portuguese). The four images were: *i*) zero, the digitized (Figure 1*b*); *ii*) degree one, the first derivative (Figure 1*c*); *iii*) degree two, the second derivative (Figure 1*d*); and *iv*) degree three, the third derivative (not shown).

Cordeiro defined this art as *arteônica* in Portuguese (or “arteonic”), writing a Concrete manifesto in 1971 (Cordeiro, 1997). According to Fabris (1997), this new computer art “not only transforms the nature of the transposed image, but also exposes it to a wider and refined fruition”. Also, Cordeiro remarked that “a second possibility, with emphasis on visual syntax and based on Concrete Art, was capable of producing interdisciplinary works based on findings in gestalt and neurology fields”. He also regarded the “computer as an instrument for changing society through its capacity to translate reality into digital form and its ability to offer developmental alternatives through simulation processes” (Fabris, 1997). Cordeiro used this particular transformation in other artworks, such as “Portrait of Fabiana” (“*Retrato de Fabiana*”, 1970), “The Woman Who Is Not B.B.” (“*A Mulher que não é B.B.*”, 1971) and “People” (“*Gente*”, 1972/1973; Velho, 1993).

Finally, the great Brazilian architect Oscar Ribeiro de Almeida Niemeyer Soares Filho (1907 - 2012), explained in Episode 3 of the Art & Mathematics series how he determined, using derivatives, the precise curves of some of his masterpieces. These include the National Congress domes, which have represented the Brazilian Chamber of Deputies and the Federal Senate in Brasilia, the Brazilian capital since 1960. For this task, he had the help of the Brazilian civil engineer Joaquim Maria Moreira Cardozo (1897 -

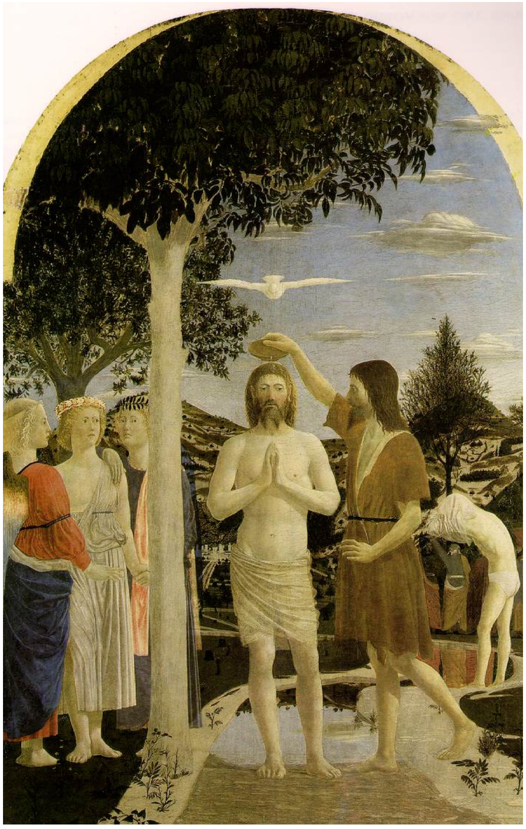
1978) (Altman, 2009). In 1987, Brasilia was declared a UNESCO World Heritage site due to its unique architecture.

Symmetry and Perspective (Episode 5)

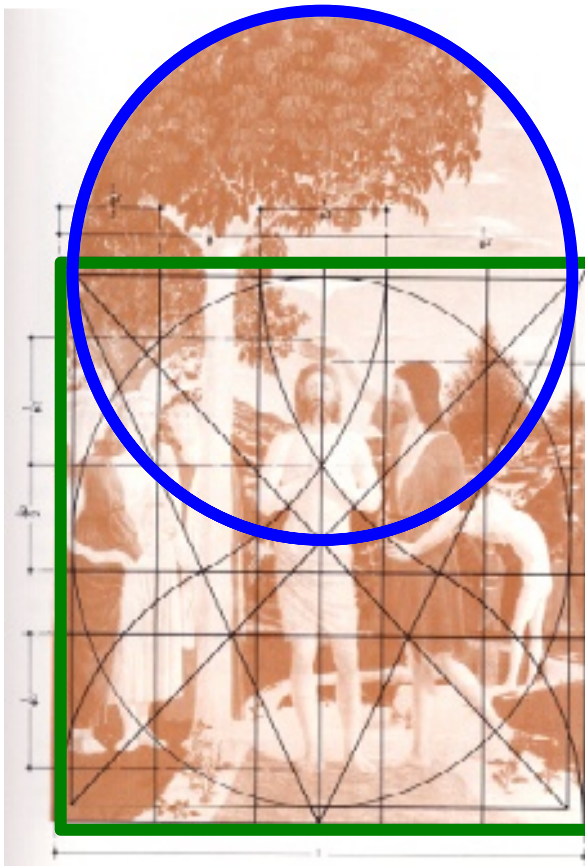
From immemorial times, primitive people have manifested their curiosity and creativity through art, dazzled by nature's patterns and geometric forms. The evolution of painting shows a clear connection with the development of mathematical ideas. For example, the notion of perspective in the Middle Ages was studied by the Italian mathematician Piero della Francesca (1415 - 1492). He showed how to better express illusions of reality on a flat surface – more specifically, false appearances. From his mathematical studies, it became possible to create a new world of events and artistic ideas.

Figure 2

The Baptism of Christ, by Piero della Francesca (1415 - 1492). National Gallery, London: www.nationalgallery.org.uk.



a)



b)

But why did artists need to study perspective at that time? One answer involves education and religion. It is important to remember that in the Middle Ages, few people knew how to read and write, and even fewer were numerate. The use of images was an intelligent and efficient way that the Church found to transmit its teachings, illustrating the feats of the greatest prophet.

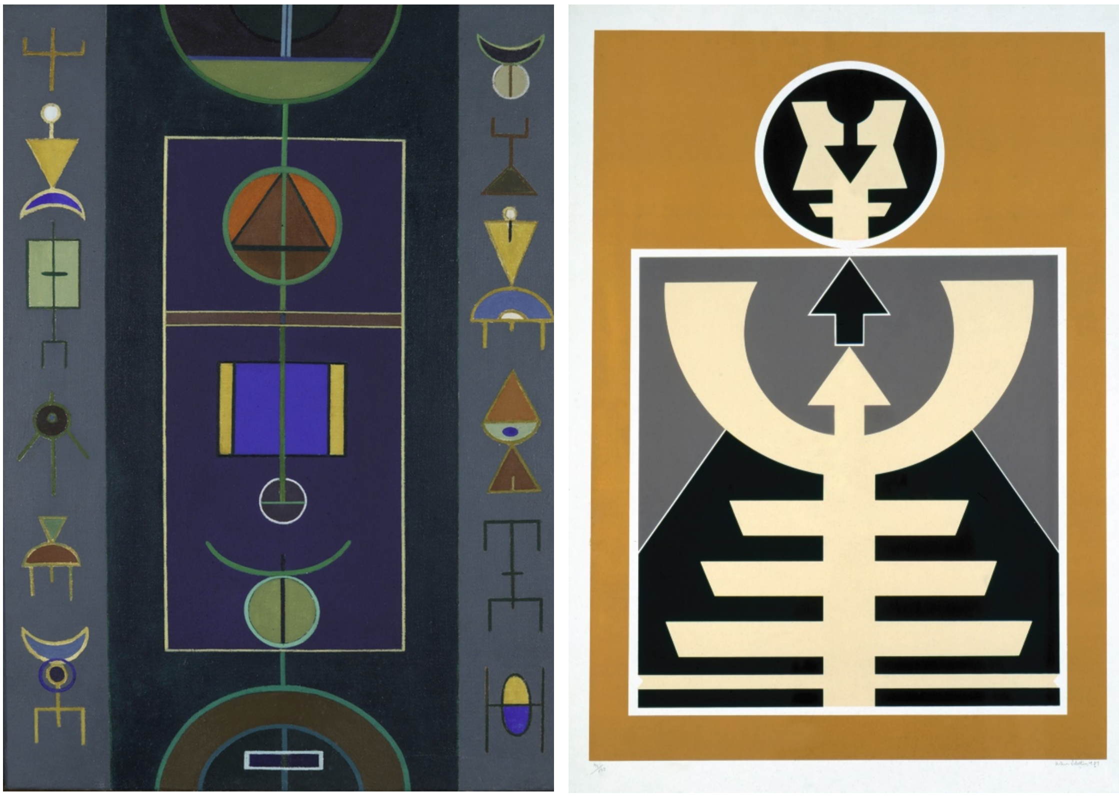
As briefly illustrated in Figure 2, della Francesca's work presented geometric and algebraic concepts. In the Middle Ages, the use of symmetry concentrated people's attention on a single *focus*: the rise of the soul, the sacred. Images were represented as celestial perfections. Following this, through its particular symbolism with the use of symmetry (including the golden ratio concept), artists sought the attention of the observer to transmit slightness and welfare (Figure 2*a*). Although there are no identical representations on both sides of the painting, the entire composition is organized around circle and square symmetries, with the first providing sky forms and the second providing land forms (Figure 2*b*). It is curious that Christ is on Earth but was inscribed in another circle.

In fact, this work has more connections with mathematical ideas: it was constructed upon the number three (see Figure 2*b*). The French illustrator and painter Charles Léon Bouleau (1906 - 1987) once noted about this masterpiece that “its breadth is divided into three, with axes falling upon the right edge of the tree and at St. John’s left side (which stretches upwards along the vertical). Its height is also divided into three, or, more exactly, into two if we merely consider the rectangular part, which has a 2/3 ratio. The semicircle on top, forming its third part, is in fact a complete circle, which one can follow along St. John’s left arm and the upper curve of the loin cloth of Christ. The dove, perfectly horizontal, remains exactly on the top of the rectangle and at the center of the circle” (Bouleau, 1963). The tree trunk divides the total width in the ratio 2/3 and the dove is placed in the same ratio to the height. Notice how the circle continues in the line of St. John’s arms.

Symmetry is still used in contemporary art. One important example, based on African sacred and cultural symbols, was proposed by the Brazilian dentist, journalist, painter, sculptor and engraver Rubem Valentim (1922 - 1991). He was unique in drawing diagrams that represented deities from Afro-Brazilian religions - known as *orishas* - such as Oshoosi’s arrow, Shango’s double-edged axe and Osanyin’s rods, which are very common in his hometown, Salvador, Bahia (see Figure 3). This could be viewed as a sensitive example of how art and mathematics have converged (Fernandes, 2017). Further studies would show how the mathematical idea of symmetry is embedded in the cultural and religious contexts of African-American sacred symbols by means of ethnomathematics (D'Ambrosio, 1985).

Figure 3

Some of Rubem Valentin's works: *a*) Untitled (1956 - 1962), oil on canvas from the Museum of Modern Art: www.MoMA.org. *b*) Untitled (1989), serigraph from the Brazilian Chamber of Deputies Collection.



a) *b*)

Math & Music (Episodes 7 and 8)

Music has long been used to express human feelings and emotions. It is interesting to note that musical scale, since the German composer and musician Johann Sebastian Bach (1685 - 1750), more precisely the well-known frequency notes **Do** – **Re** – **Mi** – **Fa** – **Sol** – **La** – **Ti** (*i.e.*, the major scale) have been changed to a geometric progression (logarithmic, chromatic or equally tempered) of ratio $\sqrt[12]{2}$, following **Do** –

Do# – **Re** – **Re#** – **Mi** – **Fa** – **Fa#** – **Sol** – **Sol#** – **La** – **La#** –**Ti** (Parker, 2009). So, when one hears a piano sonata, or jazz, or even samba, one is actually listening to numbers! Not only music but any kind of dance, with its rhythm, can be expressed in mathematical terms.

This special ratio: $\sqrt[12]{2}$ comes directly when one divides consecutive frequency notes (in Hertz, or Hz) from a chromatic scale: for example, **Mi** has 329.63 Hz, and **Fa**, 349.23 Hz; **Sol#** has 415.3 Hz and **La**, 440 Hz. Both presented the same ratio, $2^{1/12}$. Bach applied this new musical scale in his masterpiece *Das Wohltemperierte Klavier* ("The Well-Tempered Clavier") in 1722, but it is still a mystery why he chose such specific frequency notes distributed logarithmically (Nascimento, 2023).

Logarithms were discovered and published for the first time in 1614 by the Scottish mathematician, astronomer and philosopher John Napier (1550 - 1617), in his masterpiece *Mirifici Logarithmorum Canonis Descriptio* (or "A Description of the Wonderful Law of Logarithms"). In the first pages of this work, Napier established his technique of devising logarithms based upon a point moving following two progressions. In his 1619 posthumous book, *Mirifici Logarithmorum Canonis Constructio* (or "A Construction of the Wonderful Law of Logarithms") a simpler and clearer presentation was given, just asking what the relationship would be between:

- an arithmetic progression:

$$1 \qquad 2 \qquad 3 \qquad 4 \qquad 5 \qquad 6 \qquad \ldots$$

- and a geometric progression:

2	4	8	16	32	64	…

The answer is a relation between such progressions using two as the base number:

$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$	$2^6 = 64$	…

naming the exponent (from Latin *exponere*, to expose) as a logarithm or "*numeri artificialis*" ("artificial numbers"). Within a few years, logarithms were applied in astronomy, as done by the brilliant German mathematician, astronomer, and astrologer Johannes Kepler (1571 - 1630).

Music critics have compared a musical scale to mathematical precision. Another curious relationship between music and mathematics was given by Bach. One of the world's most famous compositions, "*Ave Maria*", also called "Angelical Salutation", has an interesting musical structure. It was recorded in the Latin text of the prayer of the same name and developed in 1853 from a melody of the romantic French composer Charles Gounod (1818 - 1893). This composition was inspired and specially designed to be superimposed on Prelude No. 1 in C major, BWV 846, from Book I of the above cited "The Well-Tempered Clavier". In fact, Gounod had published the composition under the title: "*Meditation sur le Premier Prélude de Piano de S. Bach*" (roughly 'Meditation on the First Piano Prelude of S. Bach') (Nascimento, 2016).

Another interesting mathematical idea is related to this sacred melody. The main theme (or core) in "*Ave Maria*" is recurrently repeated, repeated and repeated, but not in the same way, at least for a common listener. Such a melodic structure has affinities with drafts, diagrams and visual sketches that mathematicians call *fractals*. Curiously, it is the repetition of a particular theme throughout the song that mathematicians understand as *self-similarity*, repeating, repeating and repeating ... To summarize, each part of the whole is whole for the part (Nascimento, 2016).

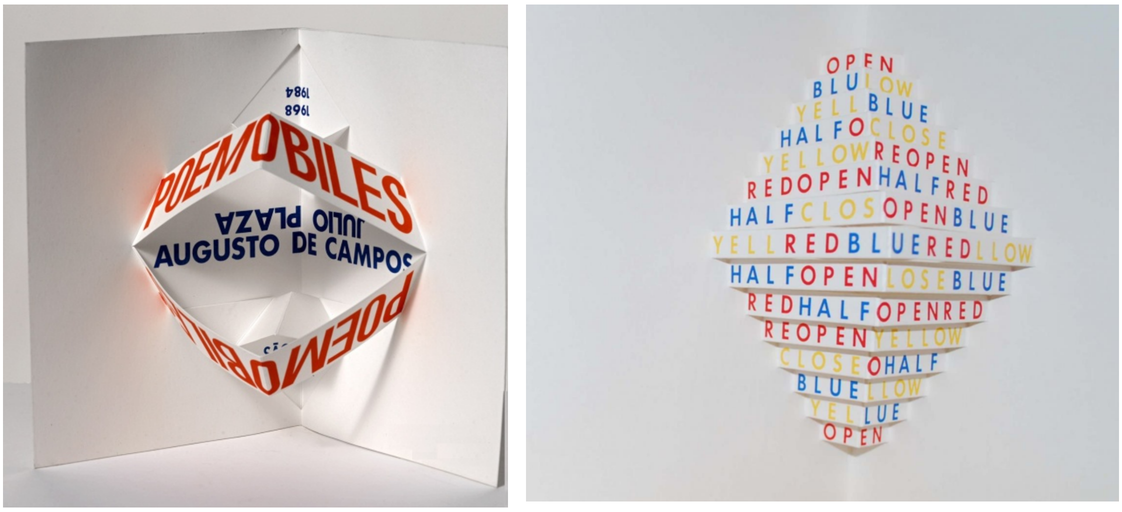
Certainly, from such example it is thus possible to hear symmetry! As another fine example, poetic elements such as rhythm, rhyme and imagery can explain why some texts and songs reach most of us (Nascimento, 2017). According to Toussaint (2013), symmetry is often cited as a contributing factor to pleasant music. He cited, for example, "*The Girl from Ipanema*", a Brazilian bossa nova song that still reaches the hearts and souls of many today. This worldwide hit was written in 1962 by the Brazilian diplomat, journalist, playwright, lyricist and poet Marcus Vinicius de Moraes (1913 - 1980) and his close friend Antonio Carlos Brasileiro de Almeida Jobim (1927 - 1994), a Brazilian composer, pianist, songwriter, arranger and singer. *Bossa nova* is a unique music genre that incorporates elements of Brazilian samba and African-American jazz.

Poetry & Topology (Episode 11)

According to the Brazilian writer, visual artist, translator and poet Augusto Luís Browne de Campos (*b*. 1931), in 1968 the Spanish artist, professor and writer Julio Plaza González (1938 - 2003) made his first "object" poem: two overlapping large cardboard pages projecting three-dimensional and colored pop-ups that formed through an interplay of cuttings and folding (Campos and Plaza, 1974). It occurred to de Campos to combine poetical texts with some of these objects written in yellow, red and blue, and so "poemobiles" were born: visual object-poems with inscribed words on several planes that displace themselves when the leaves were open, allowing projections on different planes as well as interpretations (Nascimento and Barco, 2007). The first "poemobiles" were *Abre* and Open (both have the same meaning in Portuguese and English, see Figure 4). He combined colors and words such as OPEN, CLOSE, HALF, YELLOW, RED, BLUE, to produce and evoke new results, such as HALF O CLOSE, REOPEN, BLULOW, YELLBLUE, OPENRED and LOSE BLUE. What both artists had in mind was an integrative and interdisciplinary dialogue, artistically inspired but functional, between non-figurative and figurative languages (Nascimento and Barco, 2007).

Figure 4

a) *Poemobiles* front cover; *b*) One of the first poemobiles, “Open” (detail).



a) *b*)

In the same period, the Brazilian artist Lygia Pimentel Lins (1920 - 1988), better known by her pen name Lygia Clark, gave the name *Bichos* (in Brazilian Portuguese, an animal, a critter, or a fantastic monster) to metallic aluminum leaves joined by folds and hinges that could be manipulated to produce several shapes. One can turn, fold, open and close *Bicho*, exploring its versatile nature. The hinges, joints, and layout of the metallic leaves determined an enormous set of possibilities, not all easy to note at a brief glance. Although not strictly related to mathematics, these artistic proposals could be viewed as having a mathematical (or more precisely topological) basis. For more details, see Nascimento and Barco (2007).

In brief, the first work on topology is accredited to the prolific Swiss polymath Leonhard Euler (1707 - 1783). In 1736, he published a paper on Königsberg’s Seven

Bridges problem, a set of bridges that existed in Kaliningrad, Russia. Euler analyzed ways to devise a walk through the city that would cross each of these bridges once and only once and proved that there was no solution. He analyzed the problem in abstract terms, eliminating all features except the list of land masses and the bridges connecting them. He noted that the key information was the number of bridges and the list of their endpoints, rather than their exact positions.

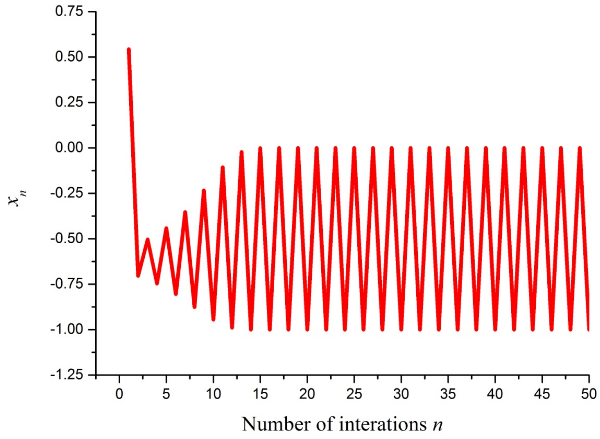
Chaos (Episode 12)

Crystals in nature present a regular and dense pattern on an atomic scale, called solid state. In two dimensions, such crystal patterns are like tessellations, or regular plane divisions, which were brilliantly illustrated by the Dutch graphic artist Maurits Cornelis Escher (1898 - 1972) in works as diverse as *Two Birds* (No. 18, 1938), *Clowns* (No. 21, 1938), *Lizard* (No. 25, 1939) and *Pegasus* (No. 105, 1959). More examples can be viewed at www.mcescher.com, site sponsored by the M. C. Escher Foundation. However, there are other simple examples, such as the famous Copacabana sidewalk in Rio de Janeiro or the Portuguese pavements in Lisbon (Hall and Teixeira, 2018).

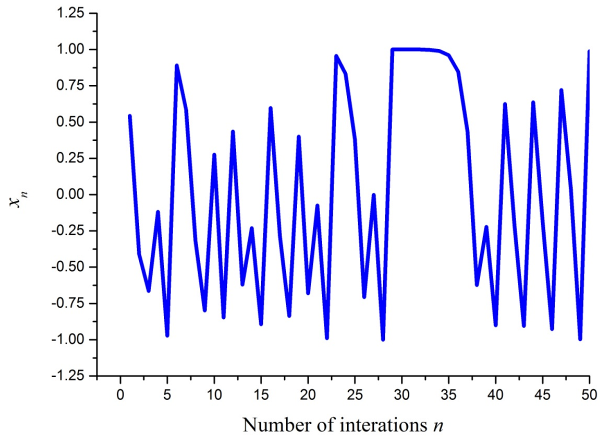
A simple picture of regular periodic potential in two dimensions related to a crystalline structure can be obtained by an elementary expression (Stewart, 1989; Nascimento, 2019) given by the British mathematician Ian Nicholas Stewart (*b*. 1945). For every positive number *n*:

Figure 5

a) Fifty iterations of $x_{n+1}= x_n^2-1$ leads to regular, periodic potential, representative of a crystalline structure, considering any starting value between $0 < x_n < 1$. In this graph, the value of x_n is plotted vertically, and iteration numbers run horizontally. The resulting values are between zero and one. *b*) Fifty iterations of $x_{n+1}= 2x_n^2-1$ leads to a non-regular potential, representative of a non-crystalline structure, considering any starting value between $0 < x_n < 1$. The resulting values are between minus one and one. This simplified representation can be viewed as an amorphous or glassy potential.



a)



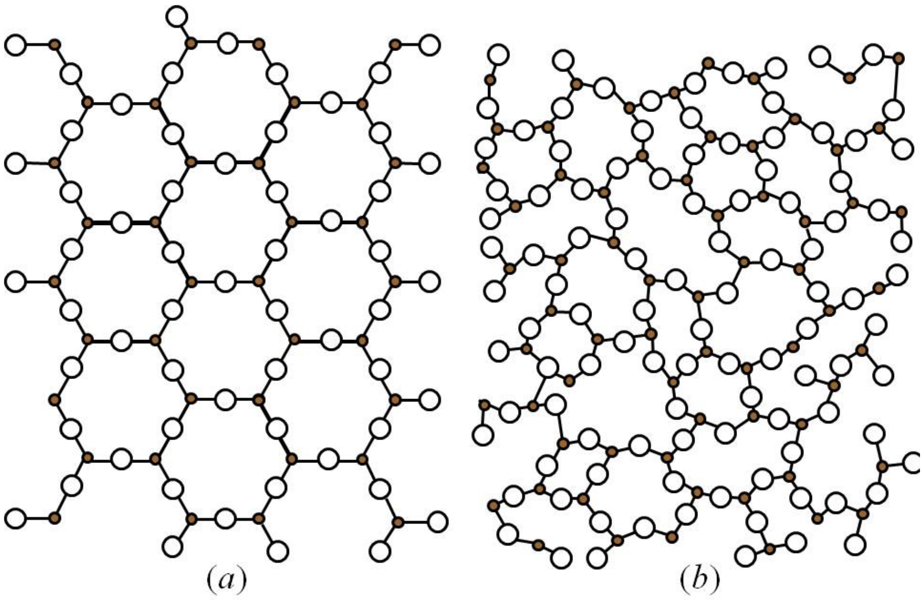
b)

$$x_{n+1} = x_n^2 - 1, \qquad \text{Eq. (1)}$$

and considering the concept of *iteration*, one can choose any number between $0 < x_1 < 1$. If $x_1 = 0.54321$ is chosen, the result is $x_2 = -0.70492$ (considering just five decimal places in Eq. (1)). Replacing this last number on the expression, iterating over and over, there is a simple pattern of zeros and minus ones (Figure 5*a*), like Bloch's potential, named after the Swiss physicist and Nobel laureate Felix Bloch (1905 - 1983), which represents a crystalline structure (Figure 6*a*). The result, which is a regular pattern, is expected because $0^2 - 1 = -1$ and $(-1)^2 - 1 = 0$. For sure, any other number with different decimals would work.

Figure 6

a) Schematics of a bidimensional molecular periodic arrangement of a crystal structure similar to B_2O_3, where B represents the small atoms and O the large atoms. Borons are coordinated to three oxygens in the crystalline phase, and each oxygen is linked to two borons. *b*) The glassy counterpart of a crystal, showing a non-periodic structure, following Zachariasen's proposal (1932).



But another simple expression, considering an integer $n > 0$:

$$x_{n+1} = 2x_n^2 - 1, \quad \text{Eq. (2)}$$

shows a different result following the same premise, *i.e.*, choosing any first number between $0 < x_1 < 1$. Starting with $x_1 = 0.54321$, the result is $x_2 = -0.40984$ (considering five decimal places in Eq. (2)). Again, iterating such results over and over, another pattern emerges (Figure 5*b*), between ones and minus ones, like Goldstein's potential, named after the American physicist Martin Goldstein (1919 - 2014). This pattern is related to a non-crystalline, amorphous or glassy structure (Figure 6*b*).

Another special result can be seen when choosing different x_1 starting values – even for small differences, such as $x_1 = 0.54322$, a resulting non-periodic pattern arises. To better visualize such concepts in terms of a regular and non-regular structure, this was presented for the first time in the classical work of the Norwegian-American physicist William Houlder Zachariasen (1906 - 1979), see Figure 6*b* (Zachariasen, 1932). He was the first to correctly propose the structure of a glassy (or non-crystalline) state.

Another interpretation was done by the American mathematician and meteorologist Edward Norton Lorenz (1917 - 2008). After simulating weather patterns by modeling a dozen variables, he discovered that small changes in the first conditions produced large changes in the long-term outcome. This gave rise to the term "butterfly effect", as written in a chapter of his book (Lorenz, 1993): "Predictability: Does the Flap of a Butterfly's Wings in Brazil Set off a Tornado in Texas?"

A simple computer experiment example was considered a new revolution in mathematics by the British-American computer scientist Stephen Wolfram (*b*. 1959) (Wolfram, 2002). According to him, traditional intuition might suggest that to do more sophisticated computations would always need advanced underlying rules. Despite the simplicity of such rules, the behavior of the iterations is far from simple, producing great complexity. What's more: due to this simplicity, no specialized scientific knowledge in physics or even about potentials or materials science is required.

Paul Jackson Pollock (1912 - 1956), the American painter, is well-known for his very large abstract panels. He created an unorthodox technique, a primitive painting style, by dripping paint from a can onto vast canvases on the floor of his windswept barn. He refined this technique by introducing different colors more or less sequentially to produce uniquely continuous trajectories. Intriguingly, some of his works, such as "*Number 14*" (1948), "*Autumn Rhythm: Number 30*" (1950) and his masterpiece "*Blue Poles: Number 11*" (1952) were partially reproduced using mathematics (Taylor *et al*., 1999; Taylor, 2002).

In the theory of simple geometry, dimension is a simple concept described by familiar integer values. For a smooth line, the dimension has a value of 1; for a completely filled area, its value is 2. However, for a fractal pattern, dimension lies between 1 and 2, as in a snowflake curve (Koch, 1906), named after the Swedish mathematician Niels Fabian Helge von Koch (1870 - 1924), or all these cited works done by Pollock. He died before chaos and fractal theories were established. Thus, he was ahead of his contemporaries in art and mathematics. Rather than mimicking art, Pollock adopted the fractal language of nature to build his own patterns.

Conclusions

We have briefly shown some examples presented in the Brazilian Art & Mathematics TV series, comprising thirteen half-hour programs. Its target audience was primary and secondary school teachers, as well as students and laymen. The aim was to cultivate a different perspective on mathematics and approach it in a new way. This series was created to show that, in many instances, History, Art, and Mathematics intermingle and that some mathematical rules are artistic.

Art and mathematics overlap because they both deal with particular activities or products made by humans for aesthetical or communicative purposes, expressing ideas over time. Poets write poems in the same way that mathematicians establish theorems.

In particular, the Brazilian mathematician, logician and philosopher Newton Carneiro Affonso da Costa (*b*. 1929) in the third episode of this series (*The Artist and The Mathematician*) said: “it can be said that the intuition of a mathematician approaches *female* intuition, noting differences that only women can see. A mathematician sees the world beyond appearances and beyond immediate forms. That’s how he (or she) makes great discoveries”.

All the episodes made reference to the earliest artifacts of civilization to contemporary artworks, presenting the wealth of universal as well as Brazilian art, culture and history. It is possible to conclude that the *focus* on educational mathematics is incorrect and must change. First, it is necessary to admit that most people do not need the basics of math that follow the Bourbarki program. Lessons such as fractals, chaos, derivatives and topology can be introduced at simpler levels for the layman. This was indeed a rare experience for any mathematician or educator.

Instead of teaching mathematics as the mere manipulation of numbers, lines and algorithms, it is both important and possible to illustrate mathematical beauty in the classroom using art. This is simply because mathematics can be viewed as the “art of abstract constructions”, and art as “the science of depicting thoughts”, both views of the Brazilian applied mathematician Luiz Carlos Pacheco Rodrigues Velho (*b*. 1956). One can argue that mathematics can be viewed as art; they are both motivated by aesthetics.

It is necessary to bring mathematics to life, a possibility is by means of comparisons, as artists do. Both mathematicians and artists do not see the same things, they seem to know something the rest do not. They are unique. No intellectual and aesthetic life can be complete unless it includes the power and beauty of art and mathematics.

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